Devoir N° 15

I ① voir figure : Pour $x<3$; $f(x)=1-x$ f° affine

x	0	3
$f(x)$	1	-2

Pour $x\geq 3$ $f(x)=2x-8$

x	3	5
$2x-8$	-2	2

gr est affine

x	4	6
$g(x)$	0	-5

② $f(x)=g(x)$

si $x<3$; $f(x)=g(x) \iff 1-x=-\frac{5}{2}x+10$

$\iff \frac{3}{2}x=9 \iff x=\frac{18}{3}=6$ refusée car $x<3$

si $x>3$ $f(x)=g(x) \iff 2x-8=-\frac{5}{2}x+10$

$\iff \frac{9}{2}x=18$

$\iff x=4$ acceptée d'où $S=\{4\}$

③ Il y a un seul point d'intersection $A(4,f(4))$; $A(4,0)$.

II Soient $a,b\in\mathbb{R}$ avec $a<b$

$f(a)<f(b)<0$ car f croissante sur $\mathbb{R}$

$\Rightarrow 1-f(a)>1-f(b)>1$ car $X\longmapsto 1-X$ décroissante sur $\mathbb{R}$

$\Rightarrow \frac{1}{1-f(a)}<\frac{1}{1-f(b)}<1$ car $x\longmapsto\frac{1}{x}$ décroissante sur $\mathbb{R}_+$

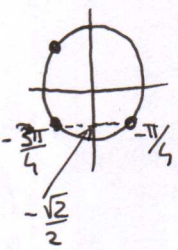
$\Rightarrow g(a)<g(b)$ donc g est croissante sur $\mathbb{R}$

III voir figure. pour ④ $\frac{93\pi}{4}=\frac{92\pi}{4}+\frac{\pi}{4}=23\pi+\frac{\pi}{4}=\pi+\frac{\pi}{4}+2k\pi$, $k\in\mathbb{Z}$

pour ⑤ $\frac{125\pi}{6}=\frac{120\pi}{6}+\frac{5\pi}{6}=20\pi+\frac{5\pi}{6}=\frac{5\pi}{6}+2k\pi$, $k\in\mathbb{Z}$

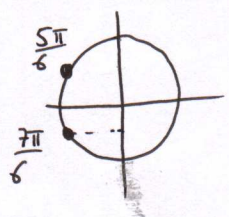
pour ⑥ $-\frac{15\pi}{3}=-5\pi=\pi+2k\pi$, $k\in\mathbb{Z}$

IV ① $\sin x=-\frac{\sqrt{2}}{2}$



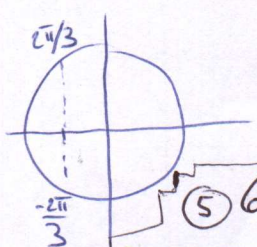
$S=\left\{-\frac{3\pi}{4};-\frac{\pi}{4}\right\}$

② $\cos x=-\frac{\sqrt{3}}{2}$



$S=\left\{-\frac{5\pi}{6};\frac{5\pi}{6};\frac{7\pi}{6}\right\}$ (ds $]-\pi;2\pi]$)

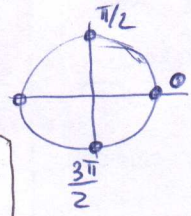
③ $\cos x=-\frac{1}{2}$



$S=\left\{\frac{2\pi}{3};-\frac{2\pi}{3}\right\}$

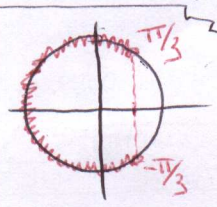
④ $\cos x\sin x=0$

$\iff \cos x=0$ ou $\sin x=0$



$S=\left\{\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi\right\}$

⑤ $\cos x<\frac{1}{2}$



$S=]-\pi;-\frac{\pi}{3}[\cup]\frac{\pi}{3};\pi]$

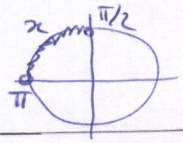
(V) $\sin x = \frac{2}{5}$.

$\cos^2 x + \sin^2 x = 1$ donc $\cos^2 x = 1 - \sin^2 x$

$= 1 - \frac{4}{25} = \frac{21}{25}$

d'où $\cos x = \frac{\sqrt{21}}{5}$ ou $\cos x = -\frac{\sqrt{21}}{5}$ mais $x \in [\frac{\pi}{2} ; \pi]$

donc $\cos x < 0$ d'où $\cos x = -\frac{\sqrt{21}}{5}$.



(VI) $(\vec{AB}, \vec{AE}) = \frac{\pi}{2} + \frac{\pi}{3} + 2k\pi$, $k \in \mathbb{Z}$

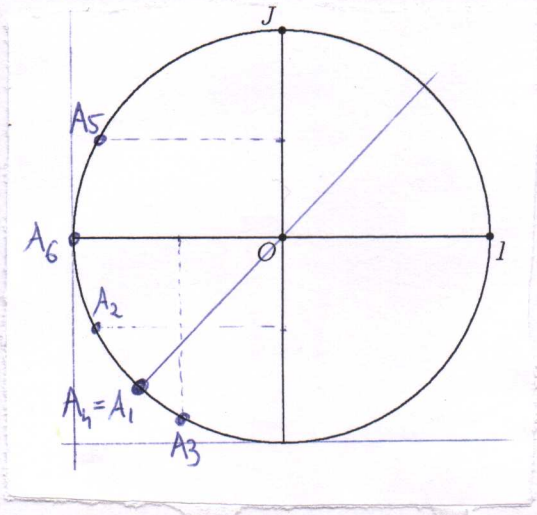
$= \frac{5\pi}{6}$; $k \in \mathbb{Z}$

$(\vec{DA}, \vec{CB}) = 0 + 2k\pi$; $k \in \mathbb{Z}$

$(\vec{ED}, \vec{EA}) = -\frac{\pi}{3} + 2k\pi$; $k \in \mathbb{Z}$

$(\vec{CA}, \vec{BA}) = (\vec{CA}, \vec{CD}) = -\frac{\pi}{4} + 2k\pi$, $k \in \mathbb{Z}$.

Annexe exercice III



Annexe exercice 1

Annexe de l'exercice 1