Devoir de Mathématiques N^o 09 (15 min) - Test calcul d'intégrales

Calculer les intégrales suivantes :

$$I_1 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + 2\sin x}\, dx$$

$$I_2 = \int_0^1 \sqrt{e^x}.\, dx$$

$$I_1 = \int_0^{\pi/2} \frac{\cos x}{1+2\sin x}\, dx = \frac{1}{2} \int_0^{\pi/2} \underbrace{\frac{2\cos x}{1+2\sin x}}_{\frac{u'}{u}}\, dx$$

$$= \frac{1}{2}\left[\ln|1+2\sin x|\right]_0^{\pi/2} = \frac{1}{2}(\ln 3 - \ln 1)$$

$$= \frac{1}{2}\ln 3.$$

$$I_2 = \int_0^1 \sqrt{e^x}\, dx$$

$$= \int_0^1 e^{x/2}\, dx = 2.\int_0^1 \underbrace{\frac{1}{2} e^{x/2}}_{u'e^u}\, dx$$

$$= 2.\left[e^{x/2}\right]_0^1 = 2.(e^{1/2} - 1)$$

$$= 2(\sqrt{e} - 1)$$

$$I_3 = \int_2^e \frac{1}{x(\ln x)^3}\,dx$$

$$I_4 = \int_{\frac{1}{3}}^{\frac{e}{3}} \frac{\sqrt{1+\ln(3x)}}{x}\,dx$$

$$I_3 = \int_2^e \underbrace{\frac{1}{x}(\ln x)^{-3}}_{u'u^{-3}}\,dx$$

$$= \left[\frac{(\ln x)^{-2}}{-2}\right]_2^e = \frac{1}{2}\left[\frac{1}{(\ln x)^2}\right]_e^2$$

$$= \frac{1}{2}\left(\frac{1}{(\ln 2)^2} - 1\right)$$

$$I_4 = \int_{1/3}^{e/3} \underbrace{\frac{(1+\ln 3x)^{1/2}}{x}}_{u'u^{1/2}}\,dx \qquad \text{et} \quad (1+\ln 3x)' = \frac{1}{x}$$

$$= \left[\frac{(1+\ln 3x)^{3/2}}{3/2}\right]_{1/3}^{e/3} = \frac{2}{3}\left[(1+\ln e)^{3/2} - (1+\ln 1)^{3/2}\right]$$

$$= \frac{2}{3}\left(2^{3/2} - 1^{3/2}\right)$$

$$= \frac{2}{3}\left(2\sqrt{2} - 1\right)$$